



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Solution by H. B. LEONARD, B. S.

From $ax^2 + by^2 + cz^2 = 0 = a\alpha x + b\beta y + c\gamma z$, $a\alpha^2 + \beta^2 b + c\lambda^2 = 1$, we get

$$x^2(a\alpha^2 + c\lambda^2)a + 2a\alpha b\beta xy + y^2(b\beta^2 + c\gamma^2)b = 0,$$

$$\frac{x}{y} = -\frac{a\alpha b\beta \pm \gamma \sqrt{(-abc)}}{a(1 - b\beta^2)}, \quad \frac{x}{z} = -\frac{a\alpha c\lambda \pm \beta \sqrt{(-abc)}}{a(1 - c\gamma^2)},$$

$$\frac{y}{z} = -\frac{b\beta c\gamma \pm \alpha \sqrt{(-abc)}}{b(1 - a\alpha^2)}.$$

Assuming $a, b, c, \alpha, \beta, \gamma$ to be real, then in order that $x : y : z$ may be real, $\sqrt{(-abc)}$ must be real. From $ax^2 + by^2 + cz^2 = 0$, it is clear that a, b, c can not all have the same sign and hence we must have one of the quantities a, b, c negative and the other two positive.

184. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

If m rows, viz., the k_1 th, k_2 th, ..., k_m th, be transferred so as to become the 1st, 2nd, ..., m th, without altering the relative positions of the remaining rows, and that n columns, viz., the k_1 th, k_2 th, ..., k_n th, be similarly transformed the determinant thus obtained is the same as the original or differs from it only in sign according as $k_1 + k_2 + \dots + k_m - \frac{1}{2}m(m+1) + k_1 + k_2 + \dots + k_n - \frac{1}{2}n(n+1)$ is odd or even. [Muir.]

Solution by G. W. GREENWOOD, B. A. (Oxon), G. B. M. ZERR, A. M., Ph. D., and H. B. LEONARD, B. S.

In transferring the p th row (or column) to the q th row (or column) there are $p-q$ interchanges of adjacent rows (or columns) and therefore $p-q$ changes of sign. Hence, in the given example, there are

$$(h_1 - 1) + (h_2 - 2) + \dots + (h_m - m) + (k_1 - 1) + (k_2 - 2) + \dots + (k_n - n),$$

$$\text{i. e., } h_1 + h_2 + \dots + h_m - \frac{1}{2}m(m+1) + k_1 + k_2 + \dots + k_n - \frac{1}{2}n(n+1)$$

changes of sign, and the determinant is unaltered in value, or differs only in sign, according as this value is *even* or *odd*; not odd or even as stated.

185. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Without introducing radicals, eliminate x and y from the equations (1) $ax^2 + bx + c = 0$, (2) $ay^2 + by + d = 0$, and (3) $ax^2y^2 + bxy + e = 0$.

I. Solution by H. F. MacNEISH, A. B., Instructor in Mathematics, University High School, Chicago, Ill., G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. E. SAUNDERS, Hackney, Ohio,

The eliminant of (1) and (3) is

$$\begin{vmatrix} a, & b, & c, & 0 \\ 0, & a, & b, & c \\ ay^2, & by, & e, & 0 \\ 0, & ay^2, & by, & e \end{vmatrix} = 0$$

or for $a \neq 0$,

$$(4) \quad ac^2y^4 - b^2cy^3 + y^2(b^2c - 2ace + b^2e) - b^2ey + ae^2 = 0.$$

The eliminant of (2) and (4) is

$$\begin{vmatrix} ac^3 & -b^2c & b^2c - 2ace + b^2e & -b^2e & ae^2 & 0 \\ 0 & ac^2 & -b^2c & b^2c - 2ace + b^2e & -b^2e & ae^2 \\ a & b & d & 0 & 0 & 0 \\ 0 & a & b & d & 0 & 0 \\ 0 & 0 & a & b & d & 0 \\ 0 & 0 & 0 & a & b & d \end{vmatrix} = 0,$$

which reduces to

$$\begin{vmatrix} a(c^2d - b^2c + 2ace - b^2e) - b^2c(b+c), & b(abc - bcd - c^2d), & -a^2e^2 \\ a & b & d \\ bc(ac + bd), & ac^2d - (b^2cd - 2acde + b^2de - a^2e^2), & be(ae + bd) \end{vmatrix} = 0.$$

II. Solution by the PROPOSER.

To avoid the introduction of determinants of high order, we proceed thus: Multiply the third equation by a and replace $a^2x^2y^2$ by $ax^2 \cdot ay^2$ obtained from the first and second.

$$\therefore b(ax + bx + c)y + bdx + ae + cd = 0.$$

Substituting in the second equation the value of y thus rationally determined, and dropping the factor a (the case $a=0$ being trivial), we obtain a second quadratic for x :

$$(a+b+d)b^2dx^2 + (2ade + 2cd^2 + bcd - abe - b^2e)bx + (ae + cd)^2 - b^2ce = 0.$$

The eliminant may now be determined in simple form.

186. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Eliminate x and y from the equations (1) $ax^3 + bx^2 + cx + d = 0$, (2) $ay^3 + by^2 + cy + e = 0$, (3) $ax^3y^3 + bx^2y^2 + cxy + f = 0$, the eliminant to be rational in d, e, f .

Solved by H. F. MacNEISH, A. B., Instructor in University High School, Chicago, Ill., and G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Using the same method as in No. 185.

GEOMETRY.

203. Additional solutions of problem 203 have been received from G. W. GREENWOOD, B. A. (Oxon) Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill., and J. CHARLES RATHBUN, A. B., Assistant in Physics, University of Washington.

205. Solutions of problem 205 have also been received from G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and G. W. GREENWOOD, B. A. (Oxon) Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.